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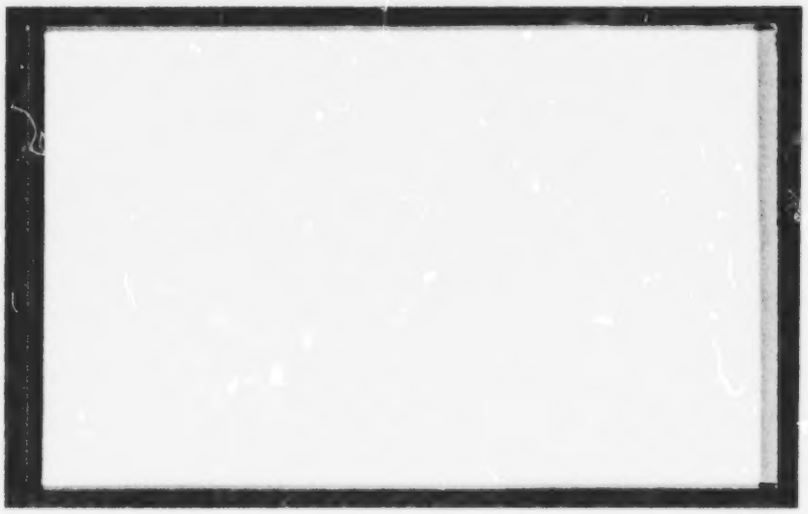

CENTRE DE RECHERCHE SUR LES TRANSPORTS

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SCHOOL BUS ROUTES GENERATOR IN URBAN SURROUNDINGS

by

Gilles Dulac, Jacques-A. Ferland
and Pierre A. Forgues

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Abstract

In this paper, a comprehensive study of the school bus routing problem in an urban surroundings is presented. It complements and extends over previous school bus routing studies. The problem is defined as a multi-stage process where the stops are generated, where the students are assigned to these, and where the bus routes are generated. Several suited versions of vehicle routing techniques are analysed and compared. Branch exchange procedures to improve the solutions are also studied. Our numerical experimentation indicates that each problem should be solved several times with different routing techniques, followed by branch exchange procedures, to ensure reliable solutions.

1. Introduction

In this paper, a comprehensive study of the school bus routing problem in an urban surroundings is presented. In general, this problem occurs in a district including several schools. The first stage of the analysis is to select the stops for each school where the students are picked up and delivered by their bus, and to assign the students to these selected stops. In the second stage, the routes are generated for each school independently. Thus, a vehicles routing problem, defined in terms of nodes to be covered, has to be solved for each school.

Several suited versions of vehicle routing techniques are analysed and compared. Notice that the routes generated with most of the standard vehicle routing techniques start and end at the same node (i.e. the school in our problem). This calls for modifications to solve our problem since a route rarely starts at the school where it ends. As a matter of fact, a prior study using crude vehicle routing techniques proved to be unsatisfactory by the school board authorities.

Section 2 includes the problem definition in term of both its characteristics and its constraints and the approach of solution. The iterative steps of the approach and the techniques are analysed in Sections 3 and 4. For the vehicle routing aspect of the problem, the following techniques are analysed: Clarke and Wright technique

[5] , [13] , Yellow technique [18] , insertion technique [17] , Gillet and Miller technique [11] , and Newton and Thomas technique [15] . To further improve the solutions, some branch exchanges procedures [14] are also studied. To conclude, the numerical results are summarized in Section 4.3 . These results indicate that a problem should be solved several times with different techniques to ensure reliable solutions. As we shall see, the "best solution" criterion is hardly (unlikely) defined because the quality of a solution depends on its easiness to be scheduled, but scheduling time scale is not included yet at this stage of the complete students' transportation planning process.

2. Problem statement

Let $G = (N,A)$ be the road network of a city. The edges are the street segments and the nodes are their intersections and the schools location. For each school in the network, the set of students to be transported there and their locations in the network are specified. Notice that even is there are many schools in the network, the routes are determined independently for each of them (or for each group of schools in the same neighbourhood considered as a unique larger school). Indeed, at this stage of the complete students' transportation process, only the length of each route and the number of students along each of them are relevant.

Our aim is to design efficient techniques to generate school bus routes subject to the following constraints:

i) the bus capacities are not to be exceeded;

ii) there is an upper bound on the transportation time;

iii) there is also an upper bound on the total distance that a student has to walk.

The approach proposed to solve this problem includes four steps:

Step 1 : Each student is assigned to a node near by his location. In general it is one of the two nodes incident to the edge where he is located in the network.

Step 2 : A set of stops is determined for each school (or group of schools). These stops are either provided by the authorities or generated by the system.

Step 3 : The students are assigned to the stops. This assignment must satisfy constraint iii) .

Step 4 : A set of routes is determined for each school (or group of schools) independently. These routes verify constraint i) and ii) . To make it easier, constraint ii) is replaced by an upper bound on the number of stops and by an upper bound on the length of each route.

This approach was first proposed by Bodin and Berman in [3] . In Step 1, the students of each school in the network are assigned to a subset of the nodes in N . These are called mini-stops in Bodin and Berman terminology. This assignment of students to mini-stops makes it easier to compute walking distances in the network and to analyse students distribution in the network.

Next, Step 2, 3, and 4 are executed for each school (or group of schools) independently. Notice the following major differences between the implementation proposed in [3] and the one in this paper. Bodin and Berman assume that the stops are provided by the school authorities whereas they can be generated by the system in Step 2 of our implementation. Also, to determine reliable school bus routes, a set of heuristic are analysed and tested.

Recall that the routes generated are the inputs for a later stage of the complete students' transportation process where the routes and school starting times are to be scheduled to minimize the bus fleet size. Hence the easiness to schedule these routes should be the most important criterion to compare the solutions generated by these heuristics. Thus, at the present stage of the complete process, since the time scale is not yet introduced, the criterion used must reflect this forthcoming scheduling stage as much as possible. The criterion, proposed here, is a compromise between the number of routes and the sum of routes lengths. On one hand, it might be easier to schedule a large

number of short routes than a small number of long ones. On the other hand, the fleet size might increase with the total number of routes. Thus, the solution strategy is to determine several solutions with different techniques, and to select the solution minimizing the product (number of routes) · (sum of routes lengths). Several other strategies with various weights for the two factors may also be analysed, but for the purpose of our study we limit ourselves to the above stated one.

An alternate approach to determine the routes would be to consider picking up the students at their locations. Then the problem would be defined in terms of street segments to be covered. However, in an urban surroundings, the authrities prefer the first alternative since the students can walk to their stops on sidewalks. Notice that this alternate approach is likely to be adopted in a rural surroundings.

The problem was stated in terms of routes taking students to school. The converse problem of taking students from the school to their locations can be solved similarly.

3. Stops generator and stops assignment

In Step 1, the edges of the network are scanned successively, and the students on each of them are assigned to one of the nodes incident to it unless neither of them are mini-stops. In this case, the students are assigned to the nearest (to one of the incident nodes) mini-stop. To reduce the number of mini-stops, the students on an edge

are assigned to the incident node (or to the nearest mini-stop) with the largest number of students already assigned to it. If the incident nodes have the same number of students already assigned to each of them, then the choice is made at random.

The stops for each school are determined in Step 2. The authorities can specify them, or they can specify a set of potential stops and an upper bound on the walking distance from the mini-stop to the stop of each student. Now, two conflicting approaches are available. The first is to minimize the total walking distance of the students with an upper bound constraint on the number of stops. The second is to minimize the number of stops with an upper bound constraint on the walking distance for each student. The second approach was adopted because it seems easier to determine better routes with a smaller number of stops.

This problem can be formulated as a set covering problem or a multi-centers location problem [4] . Instead, the stops are determined with a procedure based on student densities around potential stops. The procedure is to choose the potential stop with the largest number of students within a walking distance to it. These students are assumed to be assigned to this stop, and the procedure is repeated with the remaining students and with the remaining potential stops. The procedure ends when all students are assumed to be assigned to a stop. This procedure is much easier to implement than techniques to solve set covering problems or multi-centers location problem [4] .

This procedure is compared with a set covering approach, and the results are summarized in TABLE 1 . For three different problems, the number of stops generated with the procedure is compared with the linear programming solution of the relaxed set covering problem and with the rounded solution. The results indicate that the number of stops generated with the procedure is within 10% of the minimal number of stops. Furthermore, if the stops, generated with the two different approaches, are used to determine routes, then the total lengths of these routes are comparable.

If the authorities are not completely satisfied with the solution, then the problem can be solved again with some stops specified by them. In this case, all the students within a walking distance to these stops are assumed to be assigned to them, and the procedure is applied with the remaining students and with the remaining potential stops.

It is worth noticing that this procedure replicates the process that the authorities use to generate the routes manually. Indeed, they route the buses through the middle of densily populated areas.

Finally, in Step 3, the students are assigned to stops. The procedure is to assign each student to the nearest stop to his mini-stop. In the case where upper bounds are specified on the number of students at some stops, if these upper bounds are exceeded when the preceding procedure is applied, then a straightforward transportation problem can be formulated and solved.

Problems	Number of stops with proposed technique	Linear programming solution	Rounded linear solution	Total walking distance		Best routes generated		
				Proposed technique	Rounded linear	Number	Total length	
							Proposed technique	Rounded linear
1	54	51.75	58	116808	112025	10	50322	51519
						11	52388	52980
2	62	57	59	137996	140086	11	57154	57172
						12	55230	57172
						13	56990	68175
3	47	43	43	86090	86393	7	50120	50169
						8	51654	53734
						9	61615	54497

TABLE 1 STOPS GENERATED

4. Routes generator

Once we know the number of students assigned to each stop for a given school (or group of schools), the bus routes are generated in Step 4 according to the following constraints:

i) the bus capacities are not exceeded;

ii) there is an upper bound on the number of stops;

iii) there is an upper bound on the length of a route.

Several suited versions of well known vehicle routing techniques are analysed and compared.

There exist two main approaches solving vehicle routing problems. They are referred to as "route first - cluster second" and "cluster first - route second" in [3] . In the first approach, a giant tour going through each stop is once determined and then broken into subtours that meet the constraints. Newton & Thomas technique [15] illustrates this approach. Conversely, the second approach clusters stops into subsets, for each of which a route satisfying the constraints exists. Many techniques are available to implement this approach, and this paper includes the following: Clarke & Wright [5] , Yellow [18] , Insertion [17] and Gillet & Miller [11] .

The analysis was run in two stages: the comparison of the results obtained with these techniques and their improvement by using branch exchange procedures between routes. The results seem to replicate those in [12] for the travelling salesman. This is not surprising since

the problem studied in this paper is, apart constraints, similar to the m-travelling salesman. In general, the best solution generated by these heuristic techniques is within 7% of the best known solution, and, using branch exchange procedure on that solution, we reach within 2% of the best known solution.

Notice that a 2-OPT type ([6] and [14]) branch exchange procedure is used to solve travelling salesman subproblems when needed.

4.1 Techniques analysed

This Section briefly summarizes the vehicle routing algorithms and their versions suited to our problem.

4.1.1 Clarke and Wright technique

The algorithm is initialized by creating one route for each stop. In the following iterations, the number of routes is reduced by combining two of them. The selection of routes to be combined is based on savings.

If it is assumed that each route starts and ends at the school (as in the original version [5]), then the saving S_{ij} for each pair of stops i, j is

$$S_{ij} = d_{1i} + d_{1j} - d_{ij} \tag{4.1}$$

where 1 is the node number of the school and d_{ij} is the distance between stop i and stop j. These savings are ordered in decreasing

order, and pairs of stops i, j are successively analysed according to this ordering. For a given pair i, j, two routes are combined into one if i and j are extreme stops in two different routes (i.e. adjacent to the school in their route), and if this new route satisfies the constraints.

In the context of school busing, it is not very realistic to assume that each route starts at the school. One way to obviate this hypothesis is to evaluate the savings as follows

$$\bar{S}_{ij} = \tfrac{1}{2} d_{1i} + \tfrac{1}{2} d_{1j} - d_{ij} .$$

$\bar{S}_{ij}$ is the saving for combining two routes with i and j as extreme stops if i and j are equidistant from the school. Hence, instead of using (4.1) in our implementation, the savings are evaluated according to the following relation

$$S_{ij} = d_{1i} + d_{1j} - \lambda d_{ij} \tag{4.2}$$

with values of λ around 2 .

Another variant to evaluate the savings is also analysed:

$$S_{ij} = d_{1i} + d_{1j} - \lambda d_{ij} + [\,|\, d_{1i} - d_{1j} \,|\, - d_{ij}] \tag{4.3}$$

In this relation, a term is added to reflect the increase in the distance to move from stop i (j) to the school when stop j (i) is imposed as

an intermediary stop. To illustrate this penalty

(4.4) $$P_{ij} = [\ |\ d_{1i} - d_{1j}\ |\ - d_{ij}]\ ,$$

assume that $d_{1i} > d_{1j}$ (i.e. $d_{1i} - d_{1j} > 0$) . The triangle inequality (assumed to be satisfied) implies that

$$d_{1j} + d_{ij} > d_{1i}\ .$$

Hence $[\ |\ d_{1i} - d_{1j}\ |\ - d_{ij} = d_{1i} - d_{1j} - d_{ij} < 0$. If $d_{1i} - d_{1j} - d_{ij} = 0$, then FIGURE 1 illustrates that stop j is along the route from stop i to the school. If $d_{1i} - d_{1j} - d_{ij} < 0$, then, as illustrated in FIGURE 2, the penalty increases (negatively) for more acute detour.

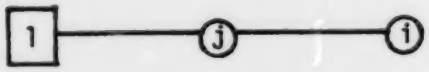
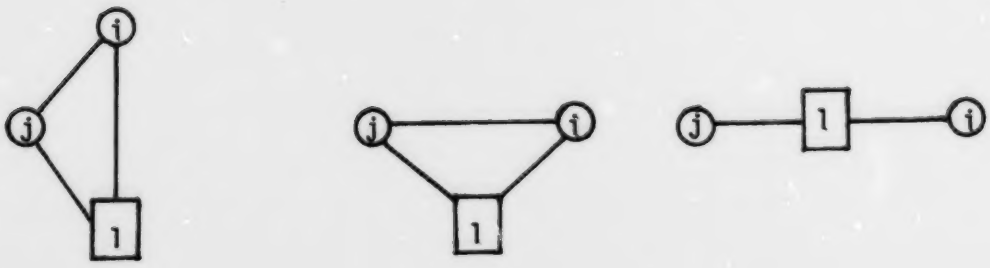


FIGURE 1



Small P_{ij} Smaller P_{ij} Smallest P_{ij}

FIGURE 2

4.1.2 Yellow technique

This technique is similar to Clark and Wright but generates only one route at a time. Given the savings evaluated with (4.2) or (4.3), the heuristic is initialized by introducing the edge (i,j) with the largest saving as the first edge on the route. Then, the route is enlarged from its ends choosing edges in decreasing order of their savings. When no other edge can be introduced to enlarge the current route, the procedure is repeated with the remaining stops to generate a new one.

4.1.3 Insertion technique

This technique also generates only one route at a time. A route is set to start at the farther stop from the school. Then, for each of the remaining stop, we evaluate the cost of introducing it between every pair of consecutive stops on the current route. The stop with the smallest cost is introduced, provided that the constraints are satisfied. This procedure is repeated with the remaining stops until no other edge can be introduced to enlarge the current route. When some stops still remains, the procedure is reinitialized to generate a new route.

Two relations to evaluate the cost A_{ikj} for introducing a stop k between two consecutive stops i and j are analysed. The first relation is defined as follows:

$$(4.5) \qquad A_{ikj} = \frac{d_{ik} + d_{kj}}{d_{ij}} \; d_{\ell k}$$

where $\ell = \begin{cases} i & \text{if } d_{1i} > d_{1j} \\ j & \text{otherwise.} \end{cases}$

Priority is given here to stops inducing the smallest detour and lying farther from the school. FIGURE 3 illustrates the relation.

In this example $A_{132} = \frac{(d_{13} + d_{32})}{d_{12}} d_{23}$ and $A_{142} = \frac{(d_{14} + d_{42})}{d_{12}} d_{24}$. If $(d_{13} + d_{32}) = (d_{14} + d_{42})$, then stop 3 is selected to be introduced in the route.

FIGURE 3

The second relation is defined as follows

$$(4.6) \qquad A_{ikj} = -\lambda (Bd_{1k} + d_{ij}) + d_{ik} + d_{kj}$$

where

$$B = \begin{cases} 1 & \text{if the route does not start at the school,} \\ 2 & \text{if the route starts and ends at the school.} \end{cases}$$

The similarity of relations (4.6) and (4.1) means that this variant resembles Yellow technique. In addition it allows the introduction of a new stop between any pair of consecutive stops on the route, so it should dominate Yellow technique.

4.1.4 Gillet and Miller (SWEEP) technique

In this technique, the stops are clustered into subsets by sweeping an imaginary ray centered at the school. Unfortunately some geographical constraints might be violated. Indeed, in some cases, pairs of stops cannot belong to the same subset (e.g. two stops on opposite river banks cannot belong to the same route unless there is a bridge near by crossing). This difficulty is obviated by checking route feasibility before introducing a new stop.

Finally, some "look ahead" procedures [3] are incorporated in our implementation. These procedures allow the introduction of other stops than the last one choosen.

4.1.5 Newton and Thomas technique

The first step of the algorithm generates a giant tour going through each stop and the school with the Clark and Wright algorithm by

assuming that a vehicle of infinite capacity is available. This tour is next broken into routes satisfying the constraints. Some "look ahead" procedures are also included here.

4.2 Exchanges procedures

These procedures are used to attempt reducing the total length of the routes by exchanging stops between routes. These are similar in essence to 2-OPT [6] and 3-OPT [14] procedures to improve the length of a tour for the travelling salesman. When considering the marginal gain and the large amount of computing time required to use 3-OPT type procedures, we restricted our analysis to 2-OPT type procedures. Furthermore, procedures of this type were judged sufficient for our purpose. Two different procedures are analysed: the first for the case where routes start and end at the school, and the second for the case where routes start at arbitrary stops.

The first procedure is a variant of the standard 2-OPT procedure [6] applied to a large tour obtained by placing the routes at the end of each other and spacing them by a dummy node duplicating the school. This is illustrated in FIGURE 4. An exchange takes place only if the new routes satisfy the constraints.

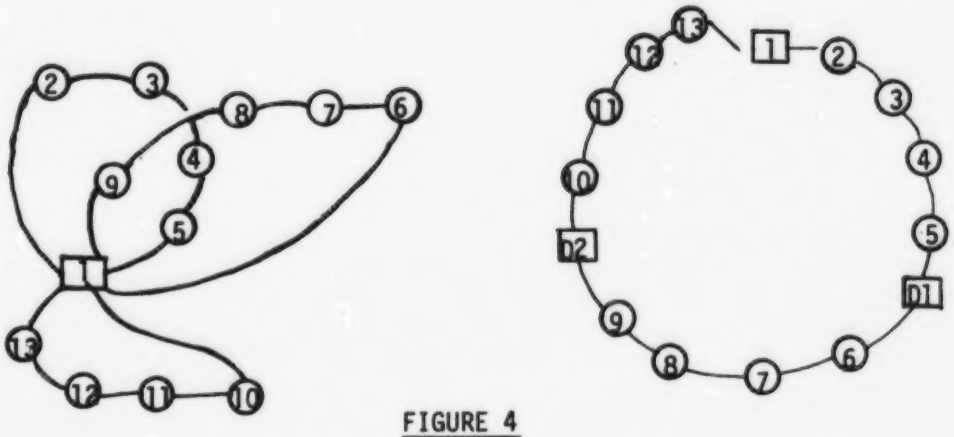
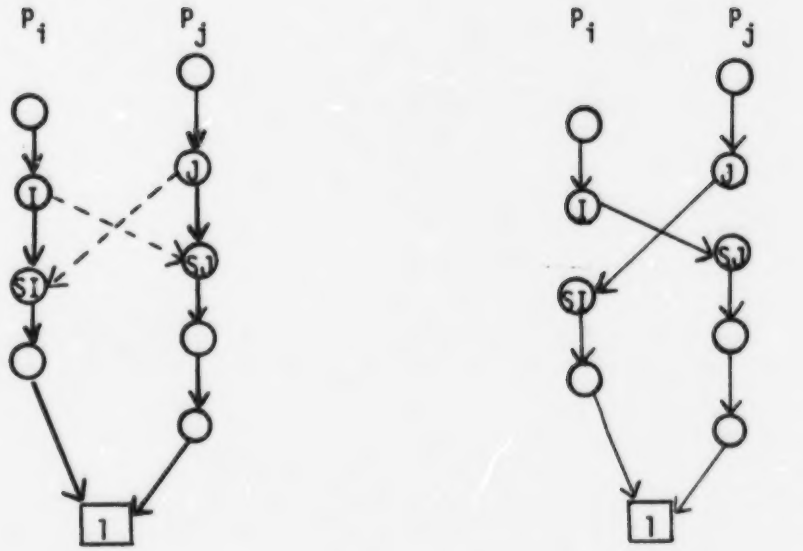


FIGURE 4

The second procedure is even more straightforward. Each pair of routes is analysed successively, and, for a given pair of routes, each possible exchange is analysed. This is illustrated in FIGURE 5. Denote the pair of routes by P_i and P_j. If, for $\Delta = 1$,

$$d_{ISJ} + d_{JSI} < \Delta [d_{ISI} + d_{JSJ}]$$

then branches $(I, SI) \in P_i$ and $(J, SJ) \in P_j$ are replaced by (I, SJ) and (J, SI) to generate the new routes P_i' and P_j' if routes P_i' and P_j' satisfy the constraints.



Before exchange

After exchange

FIGURE 5

Notice that there may exist a pair $(I, SI) \in P_i$, $(J, SJ) \in P_j$ such that the relation is not verified for $\Delta = 1$, and such that, if a 2-OPT procedure is applied to reduce the length of P_i' and P_j',

$$\text{length } P_i' + \text{length } P_j' < \text{length } P_i + \text{length } P_j .$$

Hence the test in (4.7) could be done with value of Δ larger than 1 , and 2-OPT procedure should be applied to P'_i and P'_j to verify if total length decreases.

4.3 Numerical experimentation

All these techniques and procedures have been implemented in FORTRAN on the CYBER 173 of the University of Montréal. This paper does not include any details about the computer coding, but we ought to mention that it takes advantages of the most recent data structure techniques available [10] .

The tests were performed on data made available by the Commission Scolaire Saint-François in Drummondville.

A total of 15 problems were solved with several variants of these techniques. The description of the problems (number of stops and number of students) is given in TABLES 3 and 5. TABLE 2 summarizes the experiment design specifying the techniques, their variants, and the range of the parameter λ (when necessary). The number in the last column indicates the number of time each problem is solved with different values of λ .

For this experimentation, the constraints are specified as follows. It is assumed that only one type of bus with a capacity of 60 students is used. (Notice that our implementation accepts buses of different capacities.) The upper bound on the number of stops is set to

20, and the upper bound on the length of a route is 20 kilometers. Furthermore, the distance between each stop and its 10 nearest neighbours is evaluated.

Recall that, according to the discussion in Section 2, the best solution

(4.7) Minimizes {(total distance) · (number of routes)} .

TABLE 3 summarizes the results before any branch exchange procedure is applied. The solution with the smallest total length is recorded for every different number of routes and for each variant. The total C.P. solution time in seconds including the solution time for all different values of λ is also recorded.

Notice that a study to determine the best value of λ in (4.2), (4.3), and (4.6) is omitted because it was shown by Golden and al. ([12] , [13]) that each problem must be solved with different values of λ since a good value of λ for one problem might be bad for another.

We could infer from the results in TABLE 3 that, in general, INSR (4.6) dominates INSR (4.5), YLW (4.2), and YLW (4.3) (as expected from the discussion in Section 4.1.3), and that N&T dominates G&M . Hence the analysis of branch exchange efficiency is restricted to C&W, INSR, and N&T techniques. Thus, each problem was solved according to the pattern in TABLE 4, and an improvement of each solution was attempted

with a branch exchange procedure. These results, summarized in TABLE 5, indicate that the improvement with branch exchange is smaller for CSW solutions than it is for other techniques solutions. This follows from the fact that CSW technique is minimizing total length rather than locally minimizing the length of one route at a time as does INSR technique.

Finally, in TABLE 6, the best solution using only a routing technique is compared with the best solution using also a branch exchange procedure. There, the first columns summarize the results for the best technique found in TABLE 3, the middle ones summarize the results using branch exchange procedure to improve the best solution of TABLE 3, and the last ones summarize the results for the best technique found in TABLE 5. For each problem, suppose that the best solution in TABLE 5 is taken as the best known solution. Then a percentage of deviation (% OPT) is evaluated for the two other solutions retained in TABLE 6. These results indicate that 7.3% is the maximum deviation using only a routing technique and that the maximum deviation reduces to 1.7% using also a branch exchange procedure. These results replicate those of Golden and al. [12] in their study of heuristics to solve the travelling salesman. These results also indicate that the solution in the second column of TABLE 6 is the best or second best solution accordint to % OPT.

Conclusions are very hard to draw since there is no technique dominating all the others for every problem. Furthermore, the relative

quality of the solutions obtained with the different techniques seems to be problem dependent. Hence each problem should be solved with C&W, INSR, and N&T. On the other hand, since a branch exchange procedure is very time consuming and not very useful to improve solutions generated with C&W technique, its use should be restricted to the improvement of solutions generated with INSR and N&T techniques and to the improvement of the best solution using only a routing technique.

Techniques	Variant	Range of λ	Number of solutions
Clarke and Wright	(4.2)	λ = 1.5, 1.6,..., 2.4, 2.5.	11
Clarke and Wright (C&W)	(4.3)	λ = 1.5, 1.6,..., 2.4, 2.5.	11
Yellow (YLW)	(4.2)	λ = 1.5, 1.6,..., 2.4, 2.5	11
Insertion (INSR)	(4.5)		1
Insertion (INSR)	(4.6)	λ = 0.9, 1.0, 1.1	3
Gillet and Miller (G&M)			1
Newton and Thomas (N&T)			1

TABLE 2

Techniques / Problems		C&W (4.2)	C&W (4.3)	YLW (4.2)	YLW (4.3)	INSR (4.5)	INSR (4.6)	G&M	N&T
1.1 83 stops 582 students	10 Routes 11 Routes 12 Routes 13 Routes	54.907 *	55.202°	60.922 62.247	63.408 66.175	69.556	61.629 65.837	80.175	60.886
	C.P. time	6.4	6.4	25.1	19.7	6.0	17.6	3.5	5.9
1.2 80 stops 545 students	10 Routes 11 Routes 12 Routes	52.828° 54.023	52.262* 53.964	60.522 58.536	56.849 57.460 62.179	61.155	54.984	68.283	62.736
	C.P. time	6.2	6.3	20.4	19.1	5.6	17.0	3.0	3.5
1.3 68 stops 545 students	10 Routes 11 Routes 12 Routes	51.996°	50.336*	60.124 60.883	57.492	56.398	56.329	63.518	56.989
	C.P. time	5.4	5.8	17.7	20.0	4.9	13.9	2.0	3.0
1.4 77 stops 545 students	10 Routes 11 Routes 12 Routes	52.551* 55.331	53.135° 54.017	60.088 65.292 73.688	56.177 59.844 66. 684	56.890	57.852	69.414	61.211
	C.P. time	6.3	6.7	17.8	20.2	5.9	17.8	3.1	4.1
1.5 96 stops 545 students	10 Routes 11 Routes 12 Routes	56.290 57.185	56.127° 56.956	64.269 65.110 69.610	65.325 65.454	62.558	55.831*	75.421	67.416
	C.P. time	8.2	9.1	27.19	33.15	9.02	25.13	5.61	6.22

Techniques / Problems		C&W (4.2)	C&W (4.3)	YLW (4.2)	YLW (4.3)	INSR (4.5)	INSR (4.6)	G&M	M&T
2.1	11 Routes			61.896	61.022		56.010*		
94 stops	12 Routes	54.858	54.292°	66.417	64.697	61.991			66.269
528 students	13 Routes			70.043	67.942				
	14 Routes							75.112	
	C.P. time	8.3	8.5	30.3	25.6	9.3	24.7	4.4	4.4
2.2	11 Routes			60.824	63.084	64.747	55.731*	68.321	62.466
82 stops	12 Routes	53.689°	53.689°	61.990	63.458				
595 students	13 Routes			71.114	66.604				
	C.P. time	6.3	6.3	22.7	21.0	6.3	18.5	3.6	3.5
2.3	11 Routes			68.412	62.825	60.204°	57.567*		62.774
77 stops	12 Routes		55.245	66.234	66.957			70.591	
596 students	13 Routes	57.005	57.005	68.516					
	C.P. time	5.6	6.0	24.4	26.4	6.0	17.2	2.5	3.0
2.4	11 Routes			61.072	62.852		57.419*		62.568
85 stops	12 Routes	54.884°	56.591	64.391	68.166	64.575		70.401	
595 students	13 Routes			68.194	66.516				
	C.P. time	6.2	7.4	25.7	27.0	7.4	19.5	4.6	3.8
2.5	10 Routes								60.982*
99 stops	11 Routes	56.203°		68.071	66.039		59.516		
585 students	12 Routes	59.185	60.155	65.235	70.107	69.450	64.748	73.925	
	13 Routes			68.975	77.300				
	C.P. time	8.3	8.4	30.8	33.7	8.8	26.5	6.8	6.1

Techniques / Problems		C&W (4.2)	C&W (4.3)	YLW (4.2)	YLW (4.3)	INSR (4.5)	INSR (4.6)	G&W	N&T
3.1	8 Routes	33.662 *o	33.662 *o	42.293	40.155		39.039	45.254	43.945
60 stops	9 Routes			41.143	43.104	39.484	40.474		
418 students	C.P. time	4.3	4.4	12.1	14.4	4.2	12.0	3.6	2.7
3.2	8 Routes	33.070 *o	33.070 *o	43.466	40.330	37.032	36,735		37.508
57 stops	9 Routes	33.093	33.093	42.561	38.753		37.370	30.455	
390 students	C.P. time	4.2	4.1	12.5	14.8	4.0	11.6	7.3	2.7
3.3	8 Routes	39.369*	39.615o	42.380	44.358		40.077		41.377
55 stops	9 Routes		38.400	45.472	42.612	47.346		45.921	
391 students	C.P. time	6.1	5.4	14.8	13.3	3.7	8.9	3.8	5.7
3.4	8 Routes	39.278*	39.839o	44.510	46.674		43.664	50.944	46.864
62 stops	9 Routes		43.060	44.651	47.818		40.781		
391 students	10 Routes					48.468			
	C.P. time	6.3	8.1	21.4	15.5	4.8	13.2	3.4	3.7
3.5	8 Routes	43.350o	43.350o	50.107	47.971		42.253*	48.980	46.150
70 stops	9 Routes	42.210	43.027	52.708	48.173		43.147		
391 students	10 Routes			52.574	52.592	48.234			
	C.P. time	8.8	10.4	27.1	29.6	7.3	19.6	16.6	11.0

C.P. time in seconds, * indicates the best technique, and o the second best technique for the problem using criterion (4.7)

TABLE 3

Techniques	Variant	Range of λ	Number of Solution
Clarke and Wright (C&W)	(4.2)	λ = 1.5, 1.7, 1.9, 2.1, 2.3.	5
Clarke and Wright (C&W)	(4.3)	λ = 1.5, 1.7, 1.9, 2.1, 2.3.	5
Insertion (INSR)	(4.6)	λ = 0.9, 1.0, 1.1.	3
Newton and Thomas (N&T)			1

TABLE 4

Techniques / Problems		C&W (4.2)	C&W (4.3)	INSR (4.6)	N&T
1.1	10 Routes			50.065	59.414*
83 stops	11 Routes	54.907°	55.202		
582 students	C.P. time	29.7	27.8	39.5	12.8
1.2	10 Routes	51.283*	52.262°	54.324	60.335
80 stops	11 Routes	53.192	53.964		
545 students	C.P. time	48.2	25.7	38.0	9.6
1.3	10 Routes	51.868°	50.029*	53.217	54.606
68 stops	11 Routes		52.095		
545 students	C.P. time	24.4	34.1	33.1	9.5
1.4	10 Routes	52.551*	52.837°	55.729	59.573
77 stops	11 Routes		54.324		
545 students	C.P. time	25.9	28.3	35.5	12.7
1.5	10 Routes	56.127°	56.127°	55.484*	
96 stops	11 Routes	57.558	56.384		66.086
545 students	C.P. time	35.8	35.0	52.6	15.0

Techniques / Problems		C&W (4.2)	C&W (4.3)	INSR (4.6)	N&T
2.1	11 Routes			55.809*	
94 stops	12 Routes	54.858	54.292°		62.510
628 students	C.P. time	34.0	34.0	45.0	14.4
2.2	11 Routes	54.363*		54.579°	61.392
82 stops	12 Routes	53.326	53.326		
595 students	C.P. time	39.8	36.5	42.0	9.2
2.3	11 Routes			56.770*	61.981
77 stops	12 Routes	55.272	54.969°		
596 students	13 Routes	56.944			
	C.P. time	38.8	38.9	36.7	10.7
2.4	11 Routes			55.155*	59.640°
85 stops	12 Routes	54.883	55.892		
595 students	C.P. time	39.7	36.1	45.2	12.9
2.5	10 Routes				60.982*
99 stops	11 Routes	55.848°			
585 students	12 Routes	58.941	59.895	57.874	
	C.P. time	47.8	51.7	56.0	12.1

Techniques / Problems		C&W (4.2)	C&W (4.3)	INSR (4.6)	N&T
3.1	8 Routes	33.662*o	33.662*o	37.890	38.693
60 stops	9 Routes			38.879	
418students	C.P. time	14.5	14.5	22.8	8.6
3.2	8 Routes	30.816*o	30.816*o	33.974	36.561
57 stops	9 Routes			35.866	
390students	C.P. time	23.7	23.7	23.2	5.3
3.3	8 Routes	37.423o	36.942*	38.062	39.994
55 stops	9 Routes		38.400		
391students	C.P. time	17.5	17.6	19.7	9.2
3.4	7 Routes			41.336*	
62 stops	8 Routes	38.853o	39.695	40.701	46.863
391students	9 Routes			40.635	
	C.P. time	24.62	17.8	27.1	6.5
3.5	8 Routes	41.053o	43.349	40.515*	42.859
70 stops	9 Routes	42.209	42.270	43.001	
391students	C.P. time	30.0	26.0	34.7	17.7

C.P. time in seconds, * indicates the best technique and o indicates the second best technique for the problem using (4.7)

TABLE 5

Problems	Techniques only				Best technique followed by branch exchange				Technique followed by branch exchange			
	Best technique	N.R.	DIST.	% OPT.	N.R.	DIST.	% OPT.	RANK	Best technique	N.R.	DIST.	% OPT.
1.1	C&W (4.2)	11	54.907	-	11	54.907	-	2	N&T	10	59.414	0%
1.2	C&W (4.3)	10	52.262	2%	10	52.262	2%	2	C&W (4.2)	10	51.283	0%
1.3	C&W (4.3)	10	50.336	0.6%	10	50.029	0%	1	C&W (4.3)	10	50.029	0%
1.4	C&W (4.2)	10	52.551	0%	10	52.551	0%	1	C&W (4.2)	10	52.551	0%
1.5	INSR (4.6)	10	55.831	0.6%	10	55.484	0%	1	INSR (4.6)	10	55.484	0%
2.1	INSR (4.6)	11	56.010	0.3%	11	55.809	0%	1	INSR (4.6)	11	55.809	0%
2.2	INSR (4.6)	11	55.731	2.5%	11	54.579	0.4%	2	C&W (4.2)	11	54.363	0%
2.3	INSR (4.6)	11	57.567	1.4%	11	56.770	0%	1	INSR (4.6)	11	56.770	0%
2.4	INSR (4.6)	11	57.419	4%	11	55.155	0%	1	INSR (4.6)	11	55.155	0%
2.5	N&T	10	60.982	0%	10	60.982	0%	1	N&T	10	60.982	0%
3.1	C&W (4.2), (4.3)	8	33.662	0%	8	33.662	0%	1	C&W (4.2), (4.3)	8	33.662	0%
3.2	C&W (4.2), (4.3)	8	33.070	7.3%	8	31.355	1.7%	2	C&W (4.2), (4.3)	8	30.816	0%
3.3	C&W (4.2)	8	39.369	6.6%	8	37.423	1.3%	2	C&W (4.3)	8	36.942	0%
3.4	C&W (4.2)	8	39.278	-	8	38.853	-	2	INSR (4.6)	7	41.336	0%
3.5	INSR (4.6)	8	42.253	4.2%	8	40.515	0%	1	INSR (4.6)	8	40.515	0%

N.R. is the number of routes, DIST is the total distance, % OPT is the percentage of deviation from the best known solution, and Rank is the rank of the solution obtained with the best technique followed by a branch exchange among the solutions obtained with the techniques followed by a branch exchange.

TABLE 6

5. Conclusion

This is the initial stage of a system generating and scheduling school bus routes in time. The routes generated are input into a subsystem to be scheduled in time, given that schools starting and ending times are either fixed or to be determined by the system itself (see [7]). Further reports on these other subsystems should appear in a near future.

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